MATH 32 - GRAPHING POLYNOMIALS - EXAMPLE AND EXERCISES

Here's an example of the process of sketching a polynomial, followed by a few exercises, supplemental to Homework 5.

Example: Let's sketch a graph of $p(x) = \frac{1}{3}x^3 + 2x^2 + 3x$.

We'll begin by finding the zeros of p. To do this, we'll have to factor. First let's pull out the leading coefficient $\frac{1}{3}$. We get

$$p(x) = \frac{1}{3}(x^3 + 6x^2 + 9x).$$

Notice now that each term is a multiple of x. Factoring out x, we have

$$p(x) = \frac{1}{3}x(x^2 + 6x + 9).$$

Next we recognize that $(x^2 + 6x + 9)$ has the form $a^2 + 2ab + b^2 = (a + b)^2$, where in our case $a = x$ and $b = 3$. So

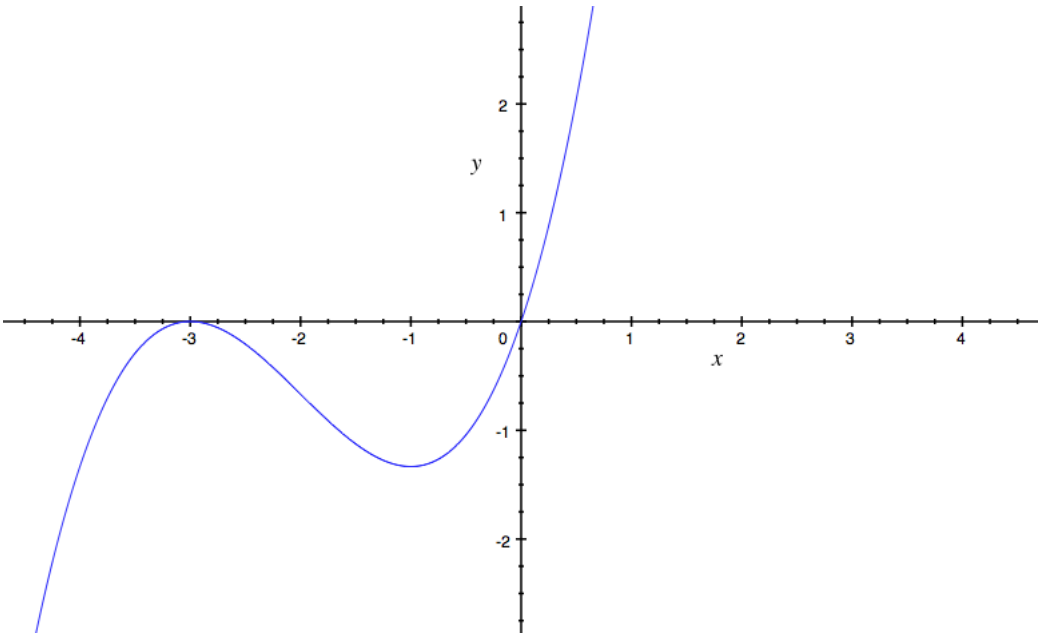
$$p(x) = \frac{1}{3}x(x + 3)^2.$$

Having completely factored p, we see that its roots are 0 and -3. Next we'll do some sign analysis to see when p is positive and negative.

	$(-\infty, -3)$	$(-3, 0)$	$(0, \infty)$
x	$-$	$-$	$+$
$(x + 3)^2$	$+$	$+$	$+$
$p(x)$	$-$	$-$	$+$

The x term is negative on the intervals $(-\infty, -3)$ and $(-3, 0)$, but it is positive on the interval $(0, \infty)$. On the other hand, $(x+3)^2$ is always positive, since it is a square! Multiplying them, we see that $p(x)$ is negative on $(\infty, -3)$ and $(-3, 0)$ and positive on $(0, \infty)$.

Now we'll sketch the graph. At this point, it could be helpful to also find the y-intercept. But for us, since 0 is a zero, the y-intercept is $(0, 0)$!



Of course, I drew this with a computer, so it's more precise than I would expect your sketch to be. In particular, there is a "valley" at about $x = -1$, and we don't have the tools in Math 32 to determine where this valley occurs, just that it must happen somewhere in the interval $(-3, 0)$.

Note how as x increases beyond 0, the value of $p(x)$ increases off the top of the window, and as x decreases below -3, the value of $p(x)$ decreases off the bottom of the window. We know that this has to happen, since for x large and positive or negative, $p(x)$ is dominated by its highest-degree term, that is, it behaves like $\frac{1}{3}x^3$.

Exercises:

For each of the following polynomial functions,

(a) Find all zeros of p.
(b) Determine the intervals on which p is positive and negative.
(c) Sketch a graph of p.

1. $p(x) = -2x^5 - x^4 + 6x^3$
2. $p(x) = (x+1)^2(x-4)^2$
3. $p(x) = x^3 - x^2 - x - 2$
   Hint: 2 is a zero of p.